



**Code-based estimate  
of the peak seismic  
demand on elastic  
structures**

**Seismic  
engineering**

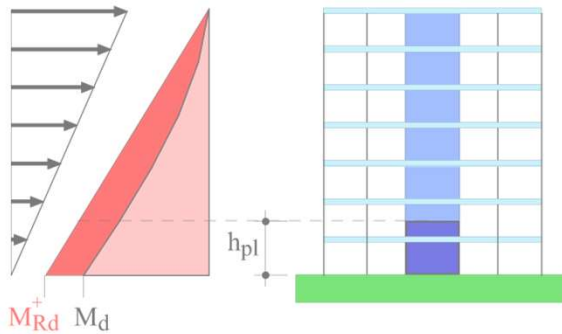
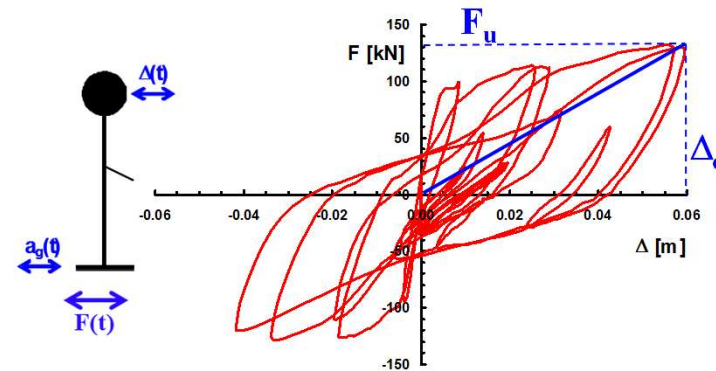
Dr. Francesco Vanin

# Course objectives

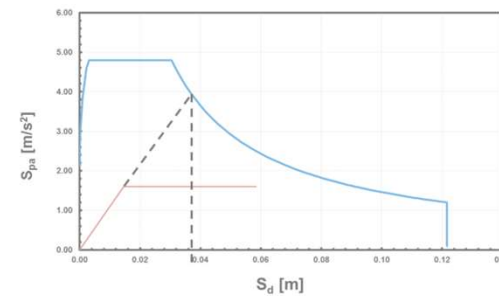
Know typical failure modes of structures during earthquakes.



Know how to estimate the peak forces and displacements of structures subjected to earthquakes.



Know how to design new buildings with reinforced concrete walls.



Know the basic elements of a displacement-based evaluation of existing structures.

# Objective and Content

## Objective:

- Know how to estimate the peak forces and displacements of elastic structures subjected to an earthquake.

## Content:

- Response spectra for a given accelerogram
- Response spectra in the code SIA 261
- Formulae for estimating the fundamental period of a building

# Elastic response spectra

# Elastic response spectra

## Definition « Response Spectra »

A response spectrum represents for SDOF systems with different periods, subjected to the same base acceleration, the maximum value of the absolute response for a certain damping coefficient.

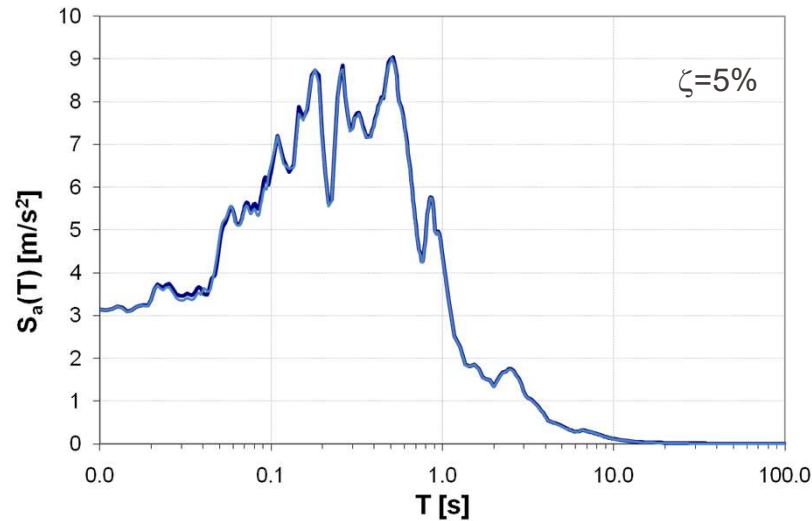
Response spectra can be calculated for many different quantities, e.g.:

- The absolute acceleration  $S_a$
- The relative velocity  $S_v$
- The relative displacement  $S_u$
- The pseudo-acceleration  $S_{pa}$
- The pseudo-velocity  $S_{pv}$

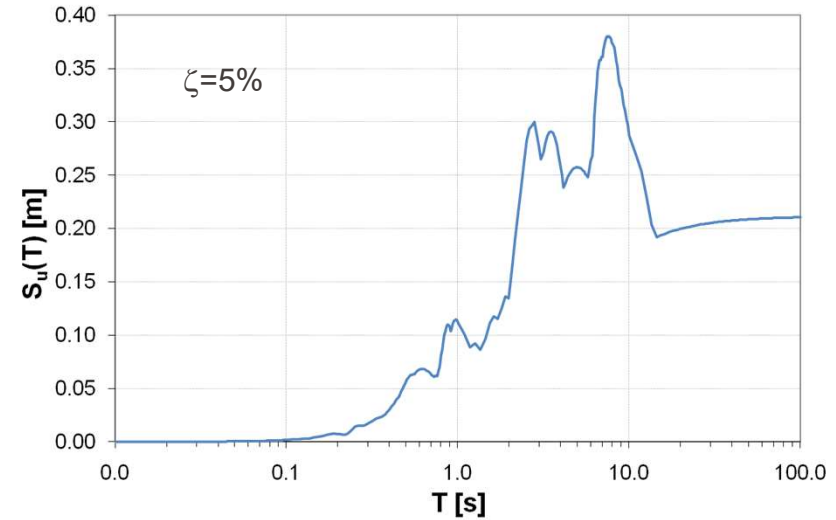
$S_u$  and  $S_{pa}$  are the most frequently used spectral values.

# Elastic response spectra

Response spectra for the  
**absolute** acceleration



Response spectra for the  
**relative** displacement



Approximate relationship between  $S_a$  and  $S_u$ :

$$S_u(\omega) \cong \frac{1}{\omega^2} S_a(\omega)$$

# One question...



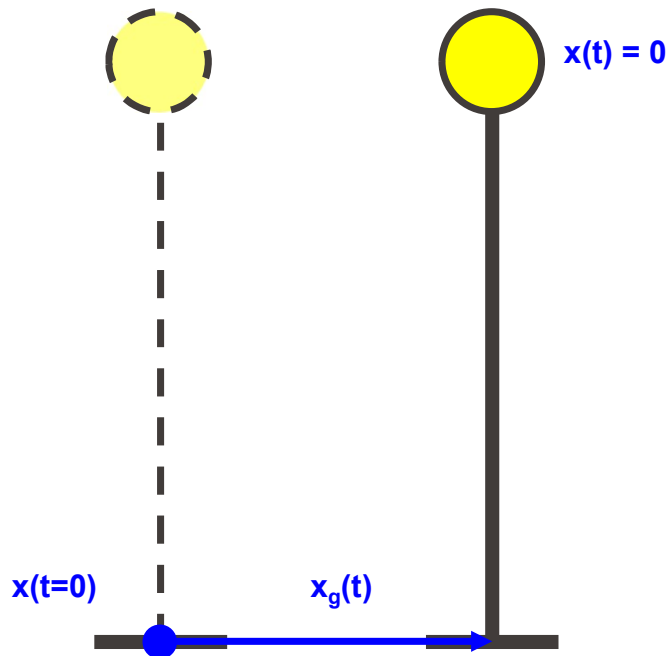
Which of the following statements is correct?

- A. For very flexible structures, the relative acceleration between mass and soil is zero and the relative displacement is not zero.
- B. For very rigid structures, the absolute acceleration is zero and the relative displacement is also zero.
- C. For very rigid structures, the absolute acceleration is zero and for very flexible structures, the relative displacement is zero.
- D. For very flexible and very rigid structures the spectral values are independent of the damping ratio

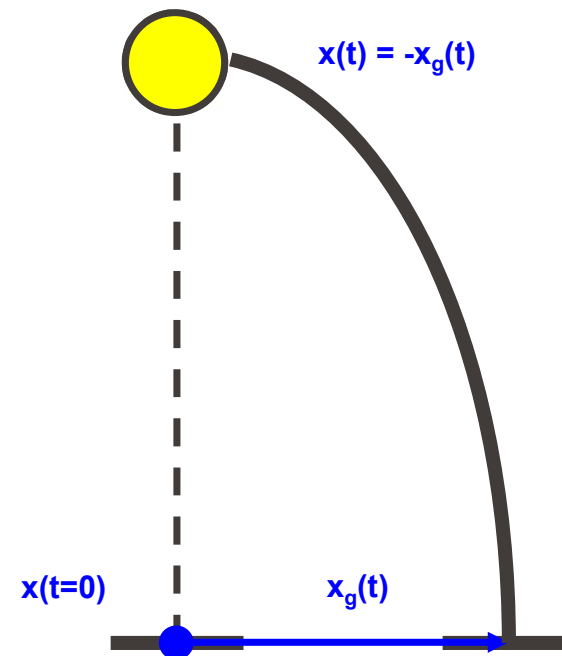


# Elastic response spectra: Limit values

Very rigid systems  
( $f \rightarrow \infty$ ,  $T \rightarrow 0$ )



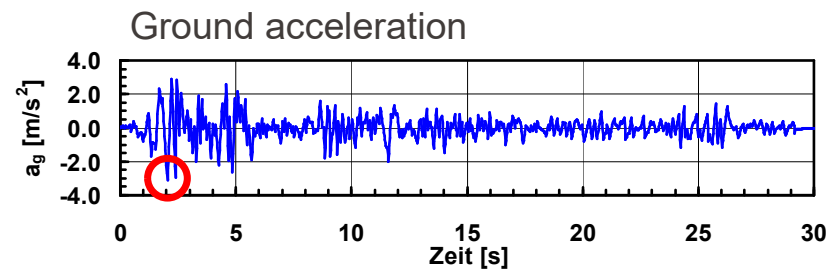
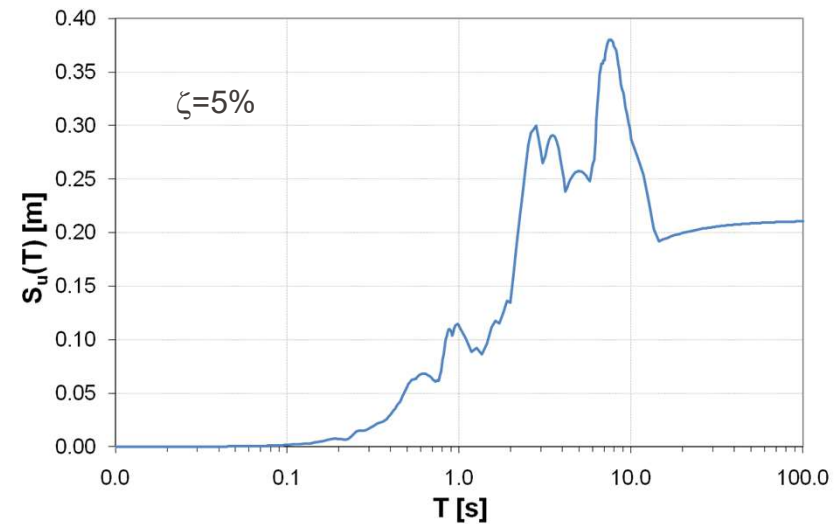
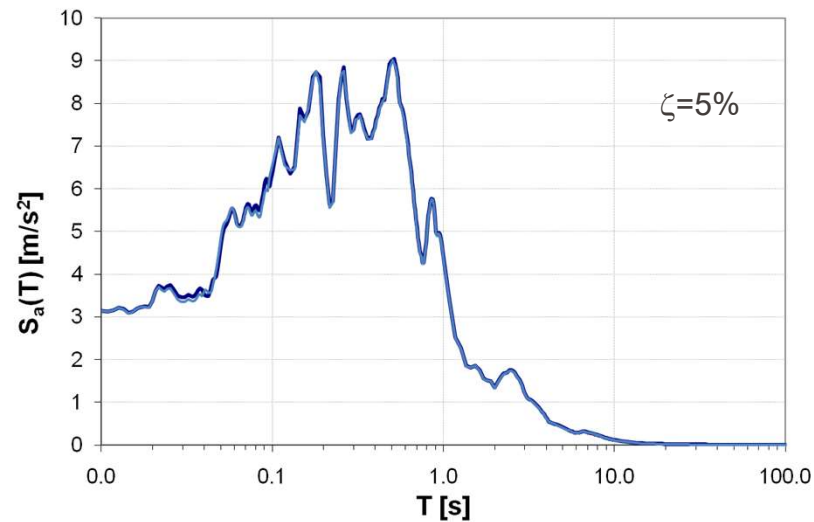
Very flexible systems  
( $f \rightarrow 0$ ,  $T \rightarrow \infty$ )



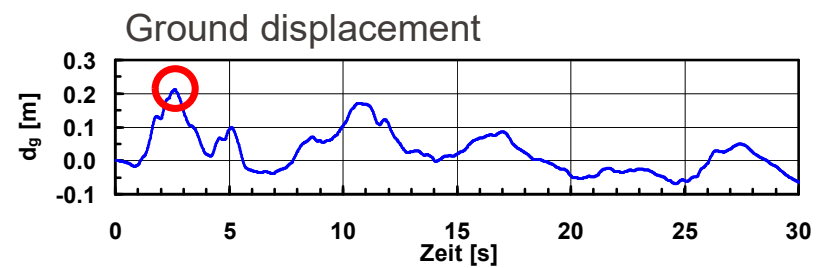


# Elastic response spectra

Spectral values for  $T \rightarrow 0$  and  $T \rightarrow \infty$



$a_{g,max} = 3.13 \text{ m/s}^2$



$d_{g,max} = 21.1 \text{ cm}$

# Elastic response spectra

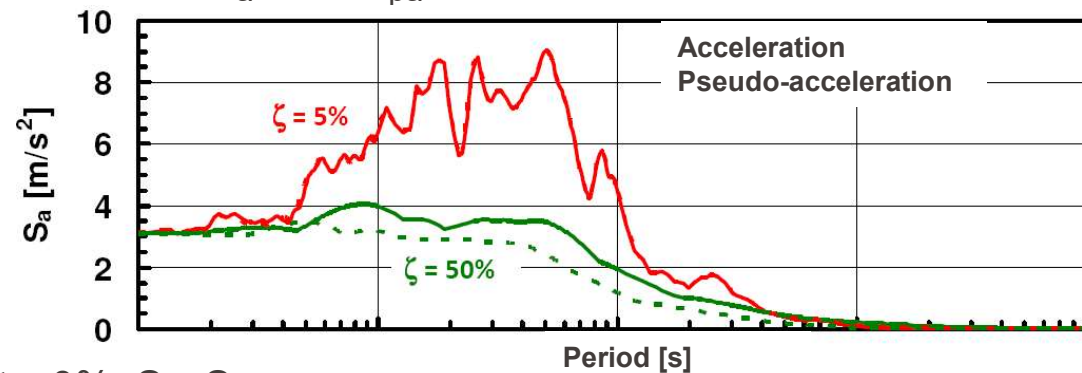
## Pseudo-acceleration $S_{pa}$

Definition of the pseudo-acceleration:  $S_{pa} \equiv \omega^2 S_u$

The pseudo-acceleration is a measure for the base shear:

$$F = kS_u = k\left(S_{pa} / \omega^2\right) = mS_{pa}$$

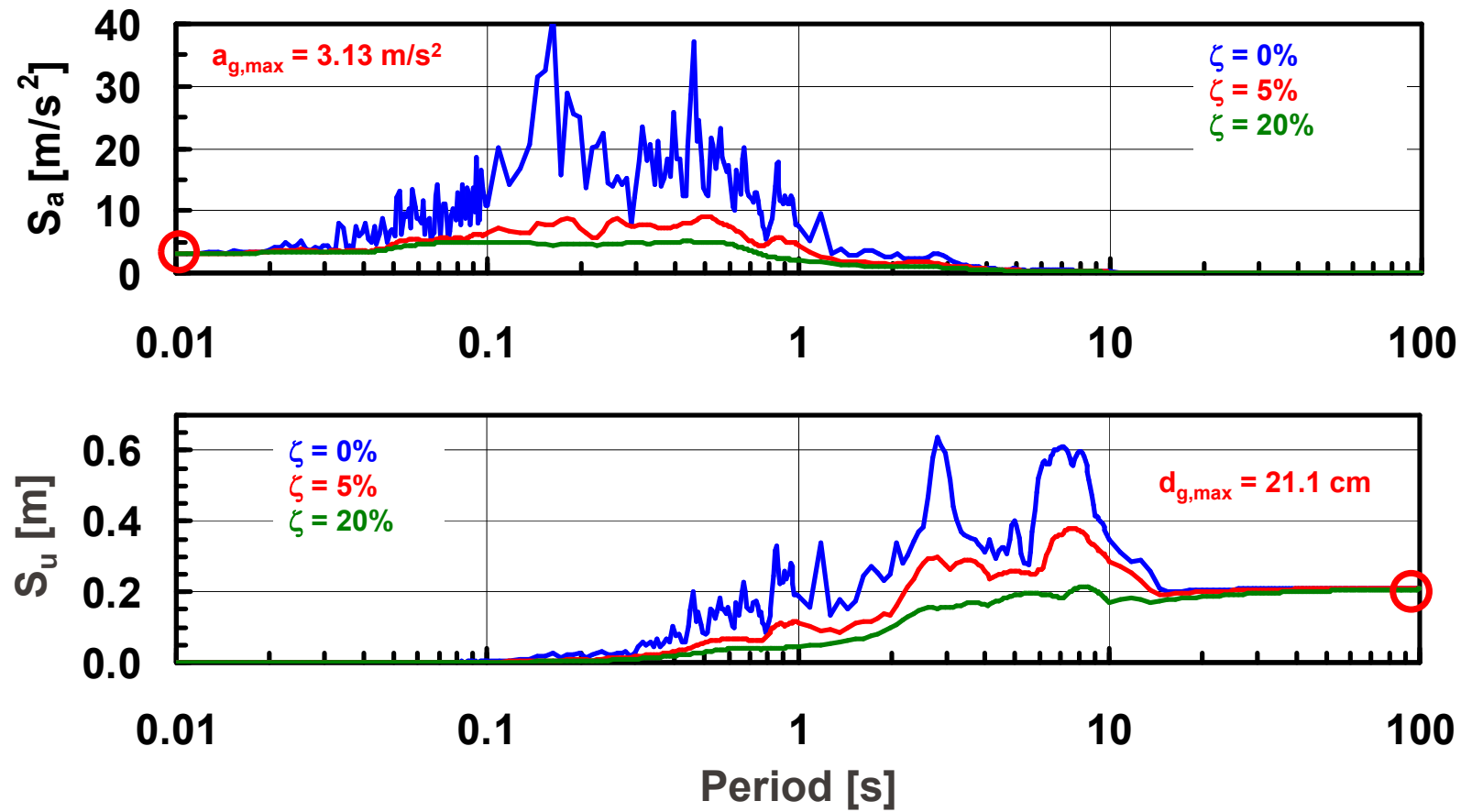
Relationship between  $S_a$  and  $S_{pa}$ :



- Damping  $\zeta = 0\%$ :  $S_a = S_{pa}$
- Damping  $\zeta < \sim 20\%$ :  $S_a \approx S_{pa}$

# Elastic response spectra

Effect of damping ratio on spectral values

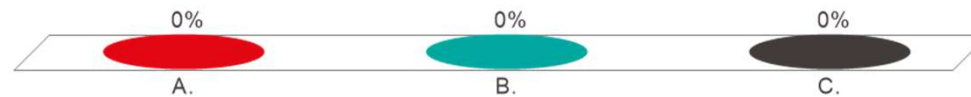
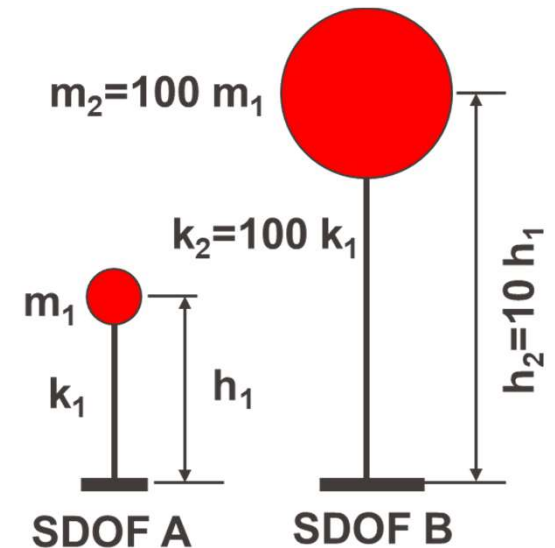




# One question...

Two SDOF systems A and B are subjected to the same ground motion. Which of the following statements is correct? Assume for both structures the same damping ratio.

- A. Both systems are subjected to the same top displacement.
- B. System B is subjected to a larger displacement at the top because it is much taller.
- C. System B is subjected to a smaller displacement at the top because it is much stiffer.

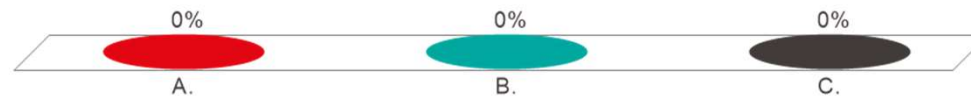
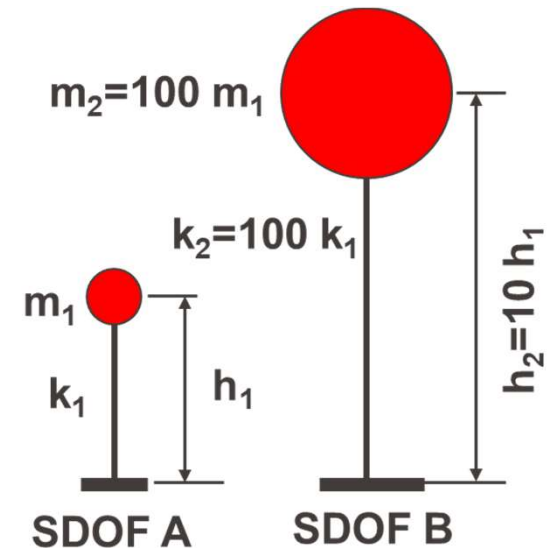




# One question...

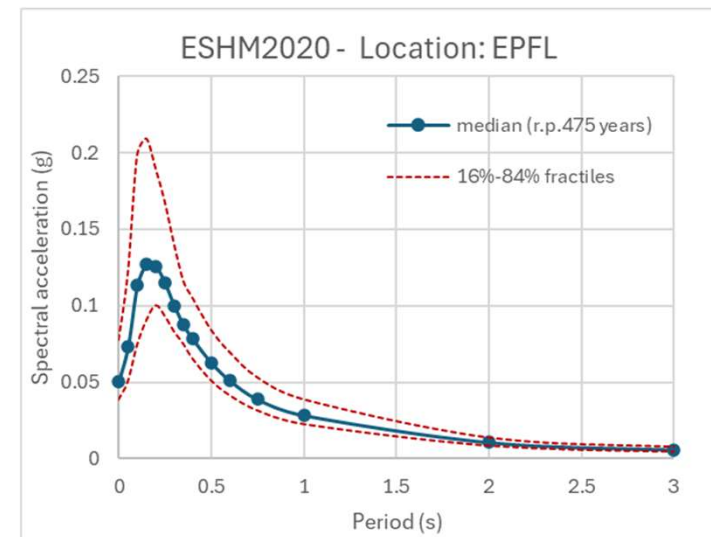
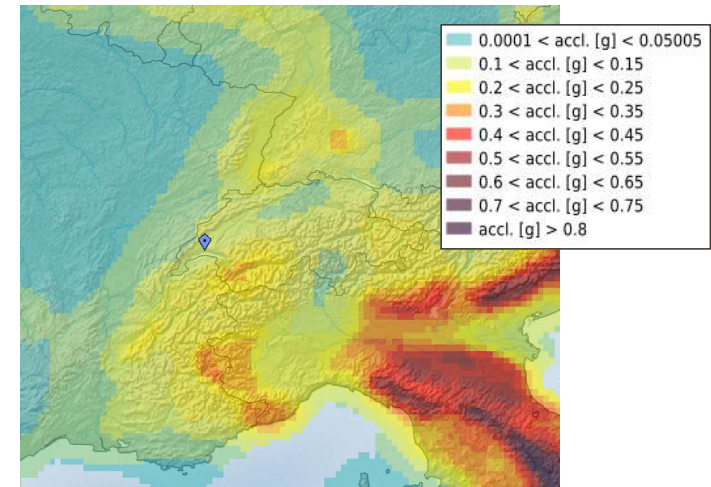
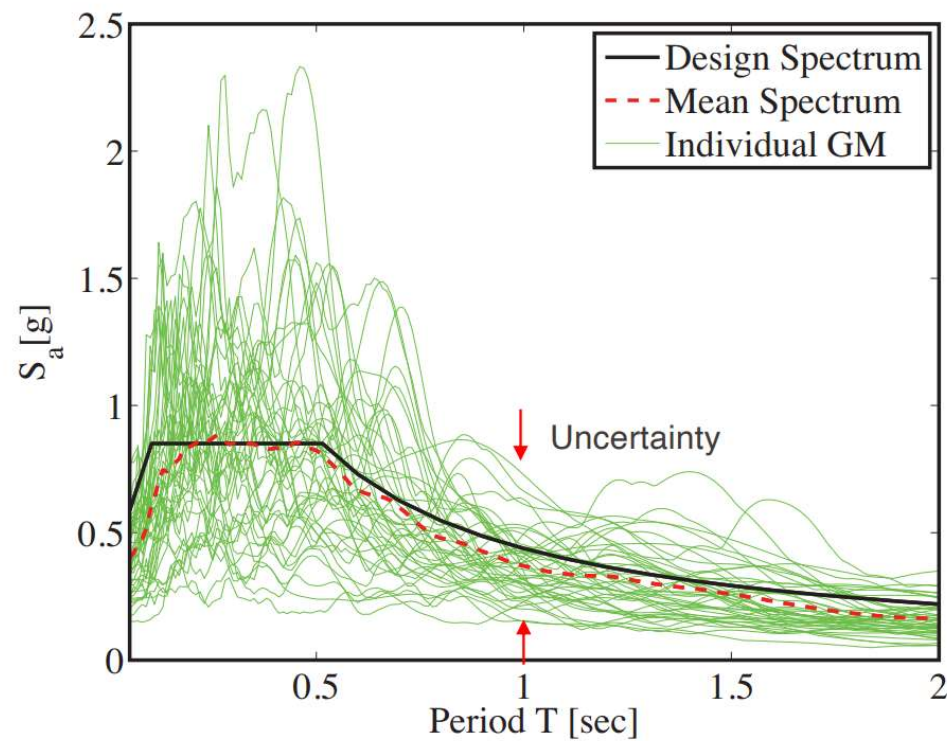
Two SDOF systems A and B are subjected to the same ground motion. Which of the following statements is correct? Assume for both structures the same damping ratio.

- A. The absolute accelerations at the top and the base shear force is the same.
- B. The absolute acceleration at the top and the base shear force of System B is larger than of System A.
- C. Both systems are subjected to the same absolute acceleration at the top, but the base shear force is not the same.



## **Elastic response spectra in the code SIA 261**

# Elastic spectra from hazard models



# Elastic response spectra in SIA 261

## Elastic response spectra in the Swiss code SIA 261

- Codes define the seismic hazard by means of response spectra.
- Spectra are defined for a return period of 475 ans → For this return period, structures should not collapse.

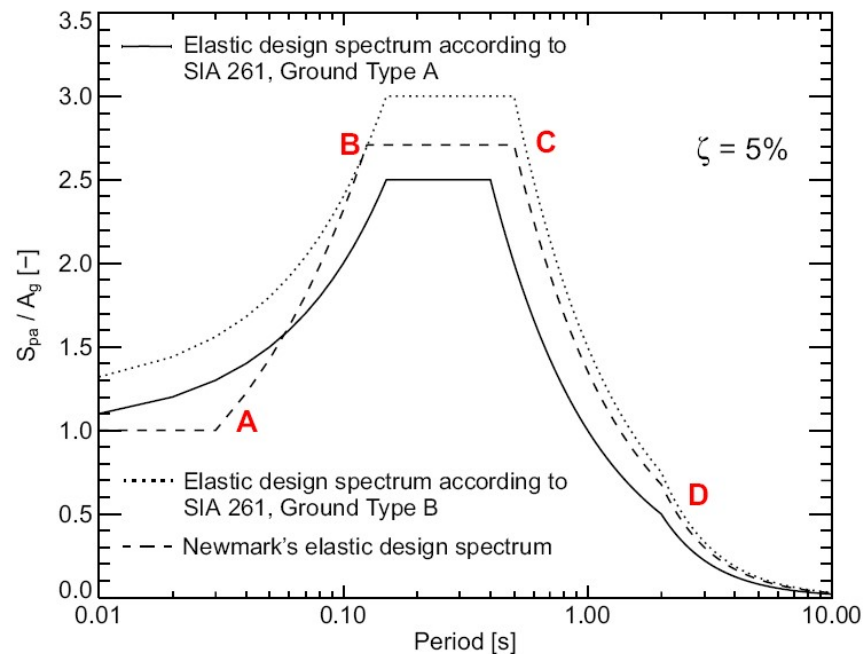
### Objective:

- Know the shape of the acceleration and displacement response spectra as defined in SIA 261
- Know the parameters which are used to define the spectra



# Elastic response spectra in SIA 261

- Elastic response spectra are typically based on Newmark spectra: Newmark N.M., Hall W.J. (1982) « Earthquake spectra and design », EERI Monograph, Earthquake Engineering and Research Institute, USA.
- Newmark spectra are smooth spectra.
- Often only three corner periods are used  $T_B$ ,  $T_C$ ,  $T_D$  (Newmark defined also a corner period  $T_A$ ).



Newmark spectra:

$T_B < T < T_C$ :  $S_{pa}$  constant

$T_C < T < T_D$ :  $S_{pv}$  constant

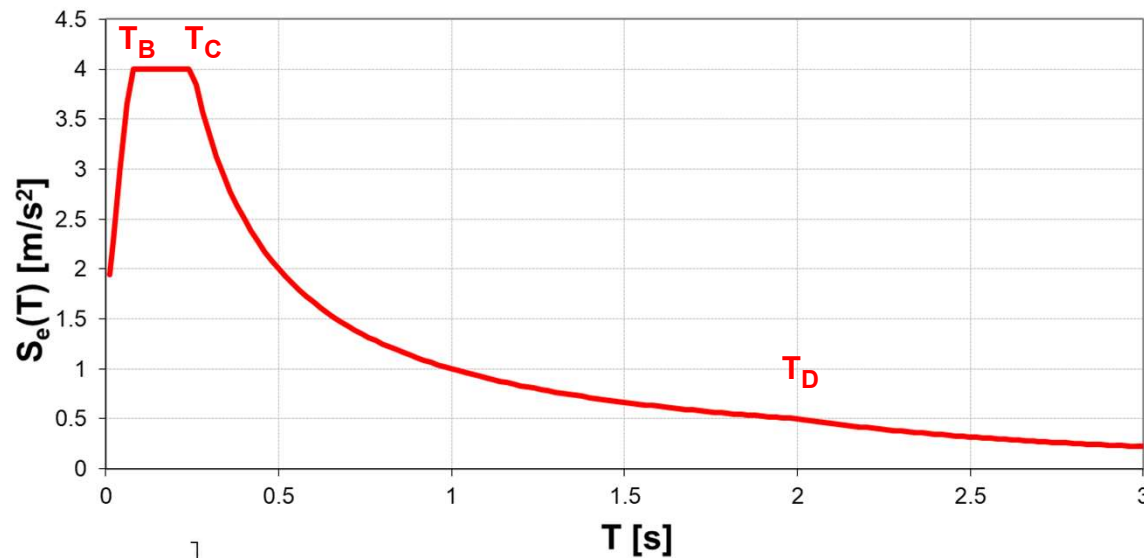
$T_D < T$ :  $S_u$  constant

# Elastic response spectra in SIA 261

- Response spectra defined for horizontal pseudo-accelerations  $S_{pa}$ .
- Two types of spectra
  - Elastic response spectrum  $S_e$  (Notation in SIA 261:  $S_e = S_{pa}$ )
  - Inelastic design spectrum  $S_d$
- The elastic response spectrum is defined as a function of
  - Seismic zone (peak ground acceleration on rock  $a_{gd}$ )
  - Soil class (corner periods  $T_B$ ,  $T_C$ ,  $T_D$  and soil parameter  $S$ )
  - Damping (parameter  $\eta$ )
- The spectrum of the vertical component of the ground motion can be estimated as 0.7 x the horizontal spectrum.

# Elastic response spectra in SIA 261

## Definition of elastic response spectrum in SIA 261



$$S_e = a_{gd} S \left[ 1 + \frac{(2,5 \eta - 1) T}{T_B} \right] \quad (0 \leq T \leq T_B)$$

$$S_e = 2,5 a_{gd} S \eta \quad (T_B \leq T \leq T_C)$$

$$S_e = 2,5 a_{gd} S \eta \frac{T_C}{T} \quad (T_C \leq T \leq T_D)$$

$$S_e = 2,5 a_{gd} S \eta \frac{T_C T_D}{T^2} \quad (T_D \leq T)$$

$a_{gd}$  = Peak ground acceleration on rock  
(dependent on seismic zone Z1a, Z1b, Z2, Z3a, Z3b)

$S$  = Parameter for soil class

$\eta$  = Correction factor to account for damping

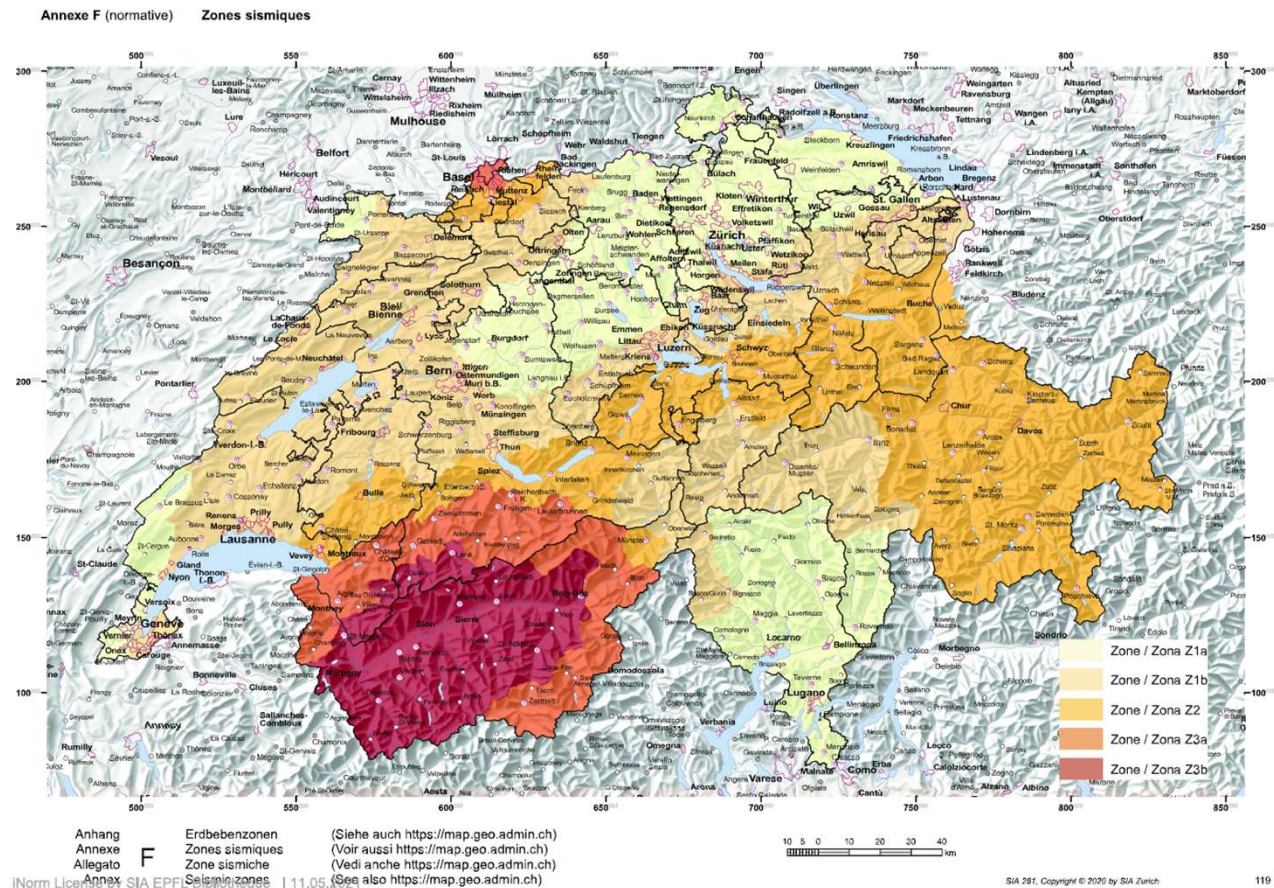
$T_B, T_C, T_D$  = corner periods (as a function of the soil class)

# Elastic response spectra in SIA 261

Five seismic hazard zones with the following peak ground accelerations on rock  $a_{gd}$  :

- Zone 1a  $a_{gd}=0.6 \text{ m/s}^2$
- Zone 1b  $a_{gd}=0.8 \text{ m/s}^2$
- Zone 2  $a_{gd}=1.0 \text{ m/s}^2$
- Zone 3a  $a_{gd}=1.3 \text{ m/s}^2$
- Zone 3b  $a_{gd}=1.6 \text{ m/s}^2$

Map available at [map.geo.admin.ch/](https://map.geo.admin.ch/)  
(<https://shorturl.at/gqsvP>)



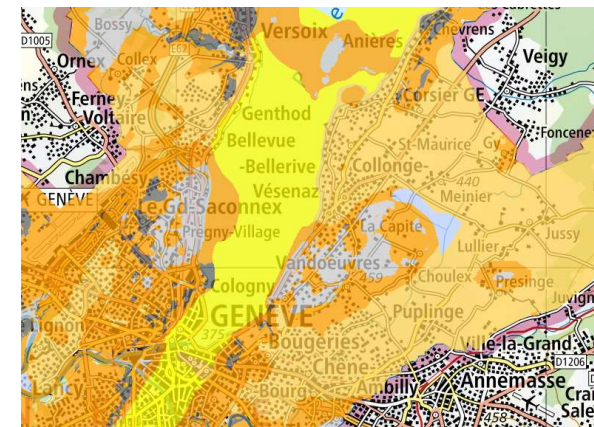
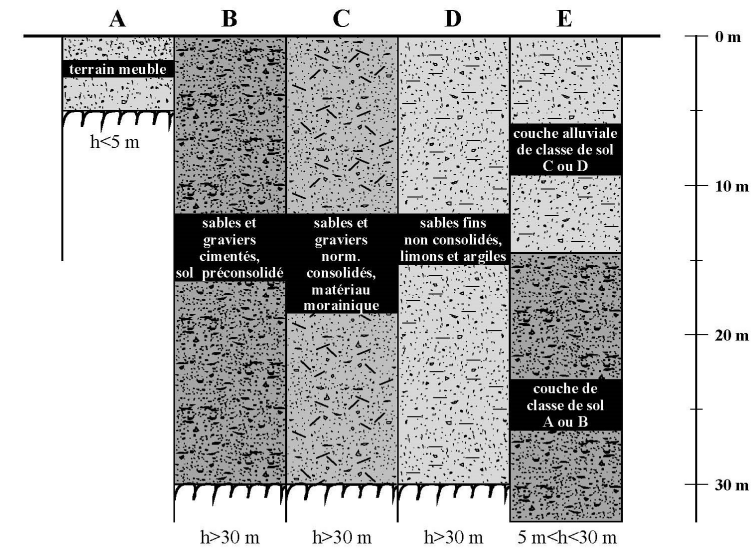


# Elastic response spectra in SIA 261

## Soil classes in SIA 261 (2020)

- 6 soil classes
- For very soft soils (class F) particular investigations are necessary

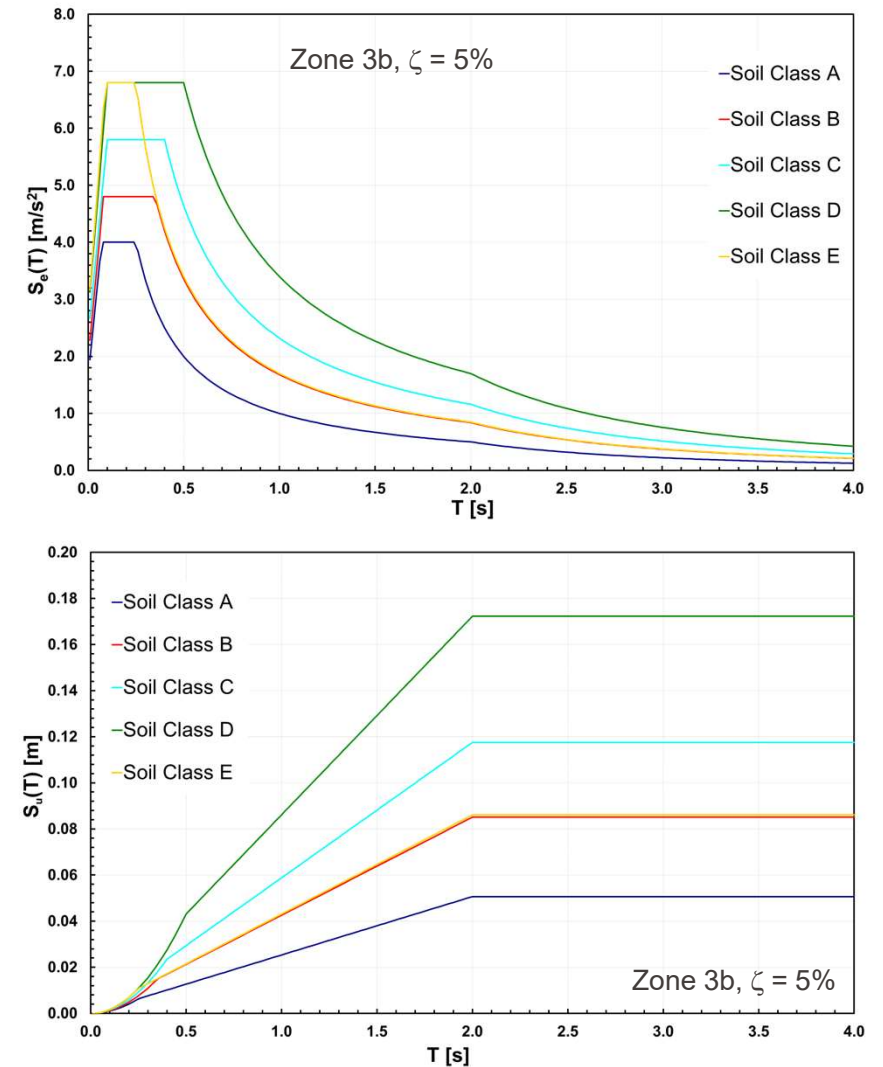
| Classe de terrain de fondat. | Description du profil stratigraphique   | $V_{s,30}$<br>m/s | $N_{SPT}$<br>nombre de coups/<br>0,3m | $c_u$<br>kN/m <sup>2</sup> | $S$  | $T_B$<br>s | $T_C$<br>s | $T_D$<br>s | $I_g$<br>m |
|------------------------------|---|-------------------|---------------------------------------|----------------------------|------|------------|------------|------------|------------|
| A                            | Rocher ou formation géologique similaire avec une couverture de terrain meuble d'au plus 5 m d'épaisseur en surface   | > 800             | —                                     | —                          | 1,00 | 0,07       | 0,25       | 2,0        | 600        |
| B                            | Dépôts de sable et gravier très compact ou d'argile très ferme, d'une épaisseur d'au moins quelques dizaines de mètres, caractérisés par une augmentation progressive des propriétés mécaniques avec la profondeur                  | 500 à 800         | > 50                                  | > 250                      | 1,20 | 0,08       | 0,35       | 2,0        | 500        |
| C                            | Dépôts de sable et gravier moyennement compact à compact ou d'argile ferme, d'une épaisseur de quelques dizaines à plusieurs centaines de mètres  | 300 à 500         | 15 à 50                               | 70 à 250                   | 1,45 | 0,10       | 0,4        | 2,0        | 400        |
| D                            | Dépôts de terrain meuble non cohésif lâche à moyennement compact (avec ou sans couches cohérentes tendres) ou à prédominance de terrain meuble cohésif de consistance tendre à ferme  | < 300             | < 15                                  | < 70                       | 1,70 | 0,10       | 0,5        | 2,0        | 300        |
| E                            | Couche superficielle de terrain meuble correspondant à la classe C ou D avec une épaisseur comprise entre 5 m et 20 m et une valeur moyenne de $V_s < 500$ m/s, reposant sur un matériau plus ferme avec une valeur $V_s > 800$ m/s | —                 | —                                     | —                          | 1,70 | 0,09       | 0,25       | 2,0        | 500        |
| F                            | Dépôts à structure sensible, organiques ou très tendres (par ex. tourbe, craie lacustre, limon mou) d'une épaisseur supérieure à 10 m   | —                 | —                                     | —                          | —    | —          | —          | —          | —          |



Map available at  
[map.geo.admin.ch/](https://map.geo.admin.ch/)  
 (https://shorturl.at/tKMY8)

# Elastic response spectra in SIA 261

Effect of soil class  
on the shape of  
response spectra



# Elastic response spectra in SIA 261

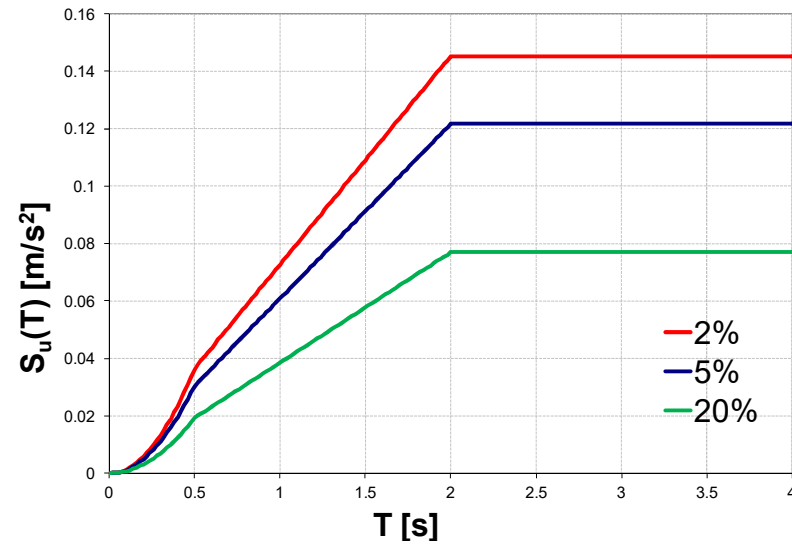
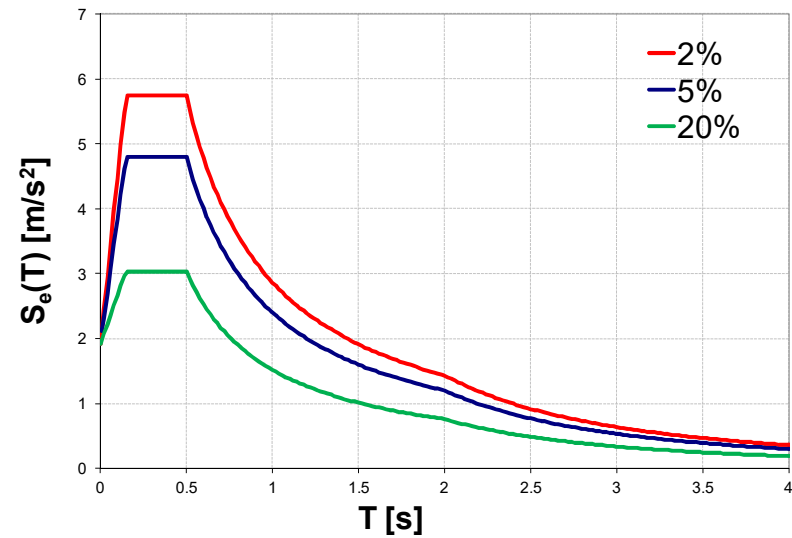
## Damping

- Elastic spectra are defined for a damping ratio of  $\zeta=5\%$
- Damping ratio depends on
  - Structural system
  - Material
  - Non-structural elements
  - ...
- For buildings: Typically  $\zeta=5\%$
- For damping coefficients different from 5% the elastic design spectra is multiplied by a factor  $\eta$  for periods  $T>T_B$ :

$$\eta = \sqrt{\frac{0.1}{0.05 + \zeta}} \geq 0.55$$

# Elastic response spectra in codes

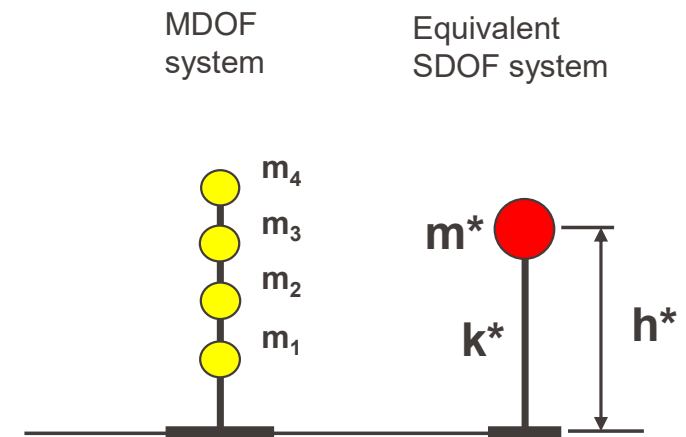
Effect of damping ratio on elastic spectra





# Estimating the fundamental period of a building

# Fundamental period of a building



- The seismic response of a building is dominated by its first mode (=fundamental mode).
- The simplest model representing the dynamic behaviour of a building is a linear system SDOF system, which has the same period as the first mode of the building.

# Fundamental period of a building

## Fundamental period of a building

- Key input parameter to code-based seismic design of structures
- Needs to be estimated at the beginning of the design process.

- Different approaches for estimating the fundamental period:
  - Empirical equations
  - Rayleigh method
  - Simplified Rayleigh method
  - Finite element calculations

# Fundamental period of a building

## Empirical equations for the fundamental period

- As a function of the number of storeys (n):

$$T_1 = \frac{n}{10} [s] \quad f_1 = \frac{10}{n} [Hz]$$

- As a function of the building height (SIA 261):

$$T_1 = C_t H^{0.75} [s]$$

- H= Building height in [m]
- Factor accounting for the structural system:
  - Steel moment resisting frames
  - RC moment resisting frames
  - All other types of structures

$$C_t = 0.085$$

$$C_t = 0.075$$

$$C_t = 0.050$$

- Equations were developed for force-based design. → To be conservative, they tend to underestimate the actual period.
- Equations were developed for structures in areas of high seismicity. → For areas of low seismicity, these equations underestimate the actual period often by large.
- It is in general not recommended to use these equations (only as very first estimate).

# Fundamental period of a building

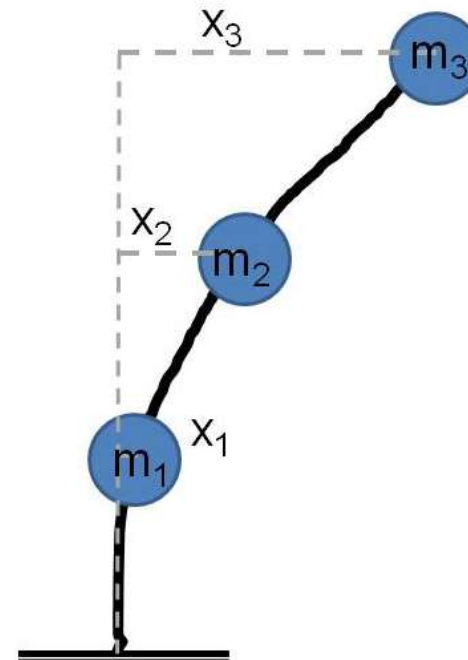
## Rayleigh method

- Method for estimating the fundamental period
- Based on the conservation of energies (potential energy = kinematic energy).
- Rayleigh quotient:

$$\omega_1^2 = \frac{x^T K x}{x^T M x}$$

- $x$  = Eigenvector of fundamental mode
- $K$  = Stiffness matrix
- $M$  = Mass matrix

→ Course “Dynamique des structures”

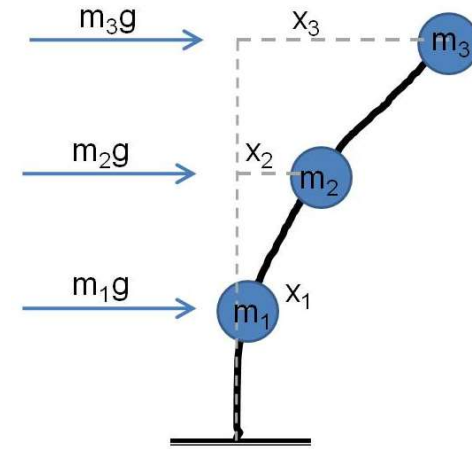


# Fundamental period of a building

## Simplified Rayleigh method (SIA 261)

- The fundamental period can be estimated as (SIA 261, Equation 40):

$$T_1 = 2\sqrt{x_{top}}$$



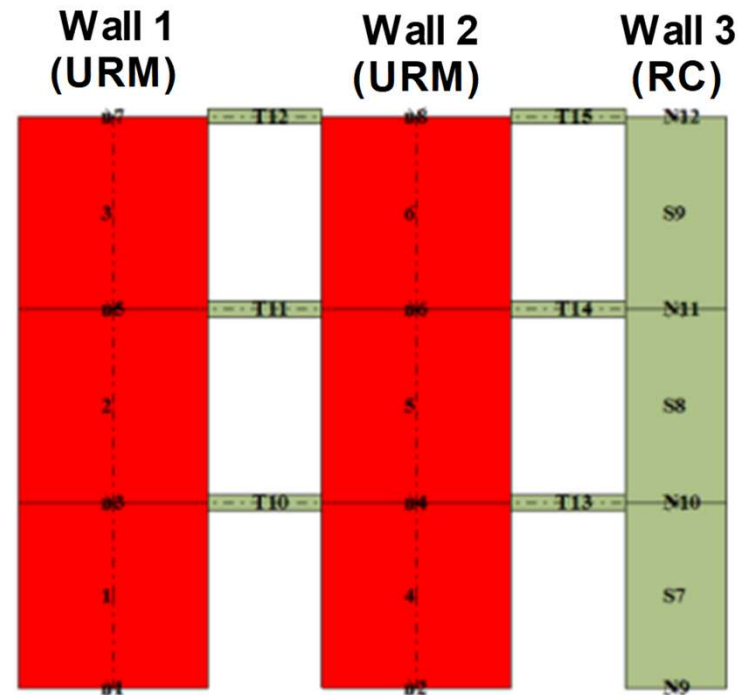
- $x_{top}$  (in m) is the fictive horizontal top displacement when the vertical loads are applied as horizontal loads.
- Vertical loads = selfweight  $G_k$  and quasi-permanent loads  $\psi_2 Q_k$
- Show that this formula is exact if applied to a SDOF system:

# Fundamental period of a building

## Finite element analysis

Numerical model of the structure with «a mean stiffness up to the point of yield» (SIA 261, 16.5.5.2)

→ typically  $EI_{\text{eff}} = 0.3-0.5 EI_{\text{gross}}$   
 $GA_{\text{eff}} = 0.3-0.5 GA_{\text{gross}}$



# References

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